Math 116 Section 04

Quiz 9 Name __________________________

August 11, 2005

Instructor: Charles Cuell Student Number __________________

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

Consider the definite integral $\int_1^2 \frac{1}{x}\,dx$.

(1) Use the trapezoid rule with $n = 4$ to estimate its value. The estimate is

$$T = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right)$$

where $y_i = f(x_i)$, $h = \frac{2-1}{4} = \frac{1}{4}$ and $x_0 = 1, x_1 = \frac{5}{4}, x_2 = \frac{3}{2}, x_3 = \frac{7}{4}, x_4 = 2$. Then

$$T = \frac{1}{8}\left(1 + \frac{8}{5} + \frac{4}{6} + \frac{8}{7} + \frac{1}{2}\right) = 0.69702381\ldots$$

(2) What value of n guarantees an error of less than 10^{-6} when using the trapezoid rule? Simpson's rule?

The error formula for the trapezoid rule is $|E_T| \leq \dfrac{K(b-a)^3}{12n^2}$, where $|f''(x)| \leq K$ on $a \leq x \leq b$. In this case, we require $10^{-6} \geq E_T$, so we solve for n in the inequality

$$\frac{K(b-a)^3}{12n^2} \leq 10^{-6}$$

For this problem, $f''(x) = \dfrac{2}{x^3}$, which is a decreasing function. Therefore, its largest value is at $x = 1$. This gives $K = f''(1) = 2$. Using $a = 1$ and $b = 2$ in the error formula and solving for n, we have

$$n^2 \geq \frac{2}{12}10^6 = 166666.666\ldots$$

n should be an integer, so round up to 166667.